

Contribution to the Workshop on Future Directions in Quark Nuclear Physics,  
 March 9-20, 1998, Adelaide. Proceedings to be published. KSUCNR-103-98  
 nucl-th/9808029

## INSIDE MESONS: COUPLING CONSTANTS AND FORM FACTORS

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We illustrate the progress of covariant QCD phenomenology for the description of meson coupling constants and form factors. As examples, we discuss the  $\rho\pi\pi$  and  $\gamma\pi\rho$  interactions, the  $\rho$  contribution to the pion charge radius, and the  $\rho NN$  and  $\omega NN$  vector and tensor coupling constants and form factors.

### 1 QCD Modeling of Mesons and Dressed Quarks

In this work the dressed quark propagators and approximation schemes are guided by the Dyson-Schwinger equation (DSE) approach<sup>1</sup> to non-perturbative QCD modeling of hadron physics. This covariant QCD phenomenology has proved to be quite efficient for low-mass mesons and their form factors<sup>2</sup>. To expedite investigations we make use of a convenient parameterization of confining solutions of quark DSEs. The broad features are taken from the solution to a simple DSE model<sup>3</sup> that is extremely infrared dominant, produces a propagator with no mass-shell pole, and includes gluon-quark vertex dressing determined by the Ward identity. The resulting propagator is an entire function in the complex  $p^2$ -plane describing absolutely confined<sup>4</sup> dressed quarks in the presence of both explicit and dynamical breaking of chiral symmetry. Additional strength for the propagator at intermediate space-like momenta is necessary to represent solutions of more realistic DSE models.

With the quark propagator written as  $S(p) = -i\gamma \cdot p \sigma_V(p^2) + \sigma_S(p^2)$ , the following parameterization for flavor  $f = u/d, s$  captures the essential features<sup>5,6</sup>

$$\bar{\sigma}_S^f(x) = \mathcal{F}(b_1^f x) \mathcal{F}(b_3^f x) \left( b_0^f + b_2^f \mathcal{F}(\Lambda x) \right) + 2\bar{m}_f \mathcal{F}(2(x + \bar{m}_f^2)) , \quad (1)$$

and

$$\bar{\sigma}_V^f(x) = \frac{2(x + \bar{m}_f^2) - 1 + e^{-2(x + \bar{m}_f^2)}}{2(x + \bar{m}_f^2)^2} . \quad (2)$$

Here  $\mathcal{F}(x) = (1 - e^{-x})/x$ ,  $x = p^2/\lambda^2$ ,  $\bar{\sigma}_V = \lambda^2 \sigma_V$ ,  $\bar{\sigma}_S = \lambda \sigma_S$  with  $\lambda$  being the mass scale. Also  $\bar{m}_f = m_f/\lambda$ , and  $\Lambda = 10^{-4}$  is not a free parameter. The five

parameters  $\bar{m}_u, b_0^u, \dots, b_3^u$  provide a good description of the pion observables:  $f_\pi; m_\pi; \langle \bar{q}q \rangle; r_\pi$ ; the  $\pi$ - $\pi$  scattering lengths; and the electromagnetic pion form factor. Kaon observables are also fit by making minimal changes to obtain the  $s$  sector quark parameters.<sup>6</sup>

The general form of the pion Bethe-Salpeter (BS) amplitude is

$$\begin{aligned} \Gamma_\pi^j(k; P) = \tau^j \gamma_5 & \left[ iE_\pi(k; P) + \gamma \cdot P F_\pi(k; P) \right. \\ & \left. + \gamma \cdot k k \cdot P G_\pi(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi(k; P) \right], \end{aligned} \quad (3)$$

and the first three terms are significant in realistic model solutions<sup>7</sup> and are necessary to satisfy the axial Ward identity.<sup>8</sup> For convenience, we employ approximate  $\pi$  BS amplitudes such as those obtained from a rank-2 separable ansatz<sup>9</sup> for the ladder/rainbow kernel of the DSE and BSE. They preserve Goldstone's theorem and should be adequate for infrared integrated quantities. Parameters are fit to  $m_{\pi/K}$  and  $f_{\pi/K}$ . The resulting  $\pi$  BS amplitude is

$$\Gamma_\pi(k, Q) = i\gamma_5 f(k^2) \lambda_1 - \gamma_5 \gamma \cdot Q f(k^2) \lambda_2. \quad (4)$$

The  $\rho$  amplitude from the same study<sup>9</sup> is

$$\Gamma_\nu(k; Q) = k_\nu^T g(k^2) \lambda_1(Q^2) + i\gamma_\nu^T f(k^2) \lambda_2(Q^2) + i\gamma_5 \epsilon_{\mu\nu\lambda\rho} \gamma_\mu k_\lambda Q_\rho g(k^2) \lambda_3(Q^2). \quad (5)$$

The BS amplitudes are normalized in the canonical way.

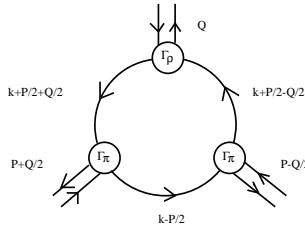


Figure 1: Diagram for the  $\rho\pi\pi$  calculation.

## 2 The $\rho\pi\pi$ Interaction

Previous attempts to explain the  $\rho\pi\pi$  coupling constant in terms of a covariant quark-gluon phenomenology for the intrinsic properties of  $\rho$  and  $\pi$  employed only  $\gamma_\mu$  and  $\gamma_5$  covariants for the respective BS amplitudes.<sup>10,11</sup> It has since been demonstrated for a number of infra-red sensitive quantities such as  $m_\pi$  and  $f_\pi$ , that the pseudovector terms in the pion BS amplitude are responsible for corrections in the 20-30% range.<sup>7,8,9</sup> Here we reexamine the  $\rho\pi\pi$  interaction with more general BS amplitudes, such as

$$\vec{\Gamma}_\pi(k; Q) = \vec{\tau} \sum_i \mathcal{K}_\pi(i) \Gamma_\pi^i(k; Q) , \quad (6)$$

for the  $\pi$ . Here  $Q$  is the  $\pi$  momentum,  $k$  is the relative  $\bar{q}q$  momentum,  $\mathcal{K}_\pi(i)$  is the  $i^{th}$  covariant constructed from gamma matrices and momenta, and  $\Gamma_\pi^i(k; Q)$  is the corresponding totally scalar amplitude. We use a parallel notation for the  $\rho$  amplitude. The first term in a skeleton graph expansion of the  $\rho\pi\pi$  vertex<sup>11</sup> yields

$$\Lambda_\mu(P, Q) = \int \frac{d^4 k}{(2\pi)^4} \Gamma_\rho^i(k'; Q') \Gamma_\pi^j(k''; Q'') \Gamma_\pi^k(k'''; Q''') T_\mu^{ijk} , \quad (7)$$

where the required discrete traces are

$$T_\mu^{ijk}(k, P, Q) = 2N_c \text{tr}_s [S(q') \mathcal{K}'_\mu(i) S(q'') \mathcal{K}''_\pi(j) S(q''') \mathcal{K}'''_\pi(k)] . \quad (8)$$

Summation over the labels (ijk) for the various meson covariants is implied. With reference to Fig. 1, the systematic notation for momenta is: the  $\rho$  vertex is characterized by single prime quantities ( $k'; Q' \equiv Q$ ), the pion vertex bringing  $P - Q/2$  into the loop is double-prime, the other pion vertex bringing  $-(P + Q/2)$  into the loop is triple-prime; the outgoing quark momentum from the prime vertex is  $q'$ , similar for the other vertices. Thus  $k' = (q' - q'')/2$ , etc. With both pions on the mass-shell,  $P \cdot Q = 0$  and  $P^2 = -m_\pi^2 - \frac{Q^2}{4}$ . In this case symmetries require the form  $\Lambda_\mu(P, Q) = -P_\mu F_{\rho\pi\pi}(Q^2)$  and the coupling constant is  $g_{\rho\pi\pi} = F_{\rho\pi\pi}(Q^2 = -m_\rho^2)$ . With the separable model BS amplitudes of Eqs. (4) and (5), the prediction for  $g_{\rho\pi\pi}$ , given in Table 1, compares favorably with the empirical value associated with the  $\rho \rightarrow \pi\pi$  decay width. Truncation to the dominant  $\rho$  amplitude is found to only make a 5% error. However the sub-dominant pion component (pseudovector  $\gamma_5 \gamma \cdot Q$ ) enters quadratically here and makes a major contribution (-71%). The calculated form factor  $F_{\rho\pi\pi}(Q^2)$  is shown in Fig. 2. Of course the detailed shape of a calculated form factor depends upon the definition of the composite meson propagator. It is always

Table 1:  $g_{\rho\pi\pi}$  calculation and contributions from meson covariants.

$g_{\rho\pi\pi} = 6.28$ [expt 6.05]	
$\pi$ Covariants	$\rho$ Covariants
$\gamma_5$	171%
$\gamma_5 \gamma \cdot Q$	-71%
	$\gamma_\mu$ 94.5%
	$\gamma_5 \epsilon_\mu \gamma k Q$ 5.5%
	$k_\mu$ 0.01%

subject to a field redefinition. We have consistently renormalized so that all momentum dependence other than that of the standard point meson propagator is allocated to the vertex function.<sup>2</sup>

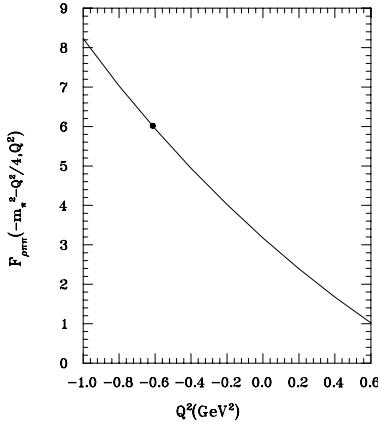


Figure 2: The  $\rho\pi\pi$  form factor versus  $\rho$  momentum. The  $\rho$  mass-shell is marked by the dot.

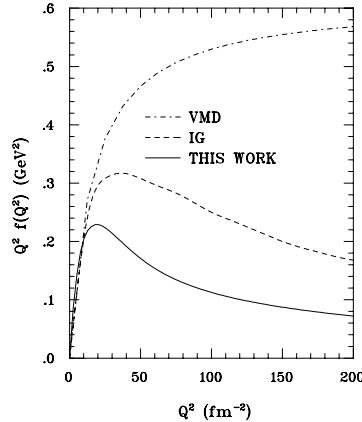


Figure 3: The  $\gamma\pi\rho$  form factor versus photon momentum.

### 3 The $\gamma\pi\rho$ Interaction

The isoscalar  $\gamma^*\pi\rho$  meson-exchange current contributes significantly to electron scattering from light nuclei. Our understanding of the deuteron EM structure functions for  $Q^2 \approx 2 - 6$  GeV<sup>2</sup> requires knowledge of this form factor.<sup>12</sup> Available studies of the vertex function  $\Lambda_{\mu\nu}(P, Q)$  use just a  $\gamma_5$  covariant for the  $\pi$  (with amplitude  $E_\pi$ ) and a  $\gamma_\mu^T$  covariant for the  $\rho$  (with amplitude  $V_\rho$ ). At the quark loop level, the expression is

$$\Lambda_{\mu\nu}(P, Q) = \frac{e}{3} \int \frac{d^4 k}{(2\pi)^4} E_\pi(k + \frac{Q}{4}; -P - \frac{Q}{2}) V_\rho(k - \frac{Q}{4}; P - \frac{Q}{2})$$

$$\times 2N_c \text{tr}_s \left[ S(k_+ - \frac{Q}{2}) \Gamma_\nu(k_+; Q) S(k_+ + \frac{Q}{2}) i\gamma_5 S(k_-) i\gamma_\mu^T \right], \quad (9)$$

where  $\Gamma_\nu(k_+; Q)$  is the photon vertex. Here  $k_\pm = k \pm \frac{P}{2}$ . The bare photon vertex  $-i\gamma_\mu$  is clearly inadequate for dynamically dressed quarks because it violates the Ward-Takahashi identity (WTI). We employ the Ball-Chiu<sup>13</sup> ansatz for  $\Gamma_\nu$  because it obeys the relevant symmetries and is conveniently determined completely in terms of the quark propagator. Then the WTI gives  $Q_\nu \Lambda_{\mu\nu} = 0$ ; the  $\gamma\pi\rho$  current is conserved. The vertex function has the general form

$$\Lambda_{\mu\nu}(P, Q) = -i \frac{e}{m_\rho} \epsilon_{\mu\nu\alpha\beta} P_\alpha Q_\beta g_{\rho\pi\gamma} f(Q^2, P^2, P \cdot Q). \quad (10)$$

The available calculation uses  $E_\pi(q; P) = B(q^2, m)/f_\pi$ , where  $B$  is the quark scalar self-energy, and also  $V_\rho(p^2) \propto e^{-p^2/a^2}$ . The norm of  $V_\rho$  is set in the canonical way. The range  $a$  is adjusted to reproduce  $g_{\rho\pi\pi}^{\text{expt}} = 6.05$ . This has proved to be a good phenomenological basis predicting other  $\rho$  processes.<sup>14,15</sup> The resulting prediction  $g_{\gamma\pi\rho} = 0.5$  agrees well with the empirical value  $g_{\gamma\pi\rho}^{\text{expt}} = 0.54 \pm 0.03$  from  $\rho$  decay. The  $\gamma^*\pi\rho$  form factor weighted by  $Q^2$  is shown in Fig. 3. The result is much softer than either the vector meson dominance (VDM) prediction or a quark loop without momentum-dependent dressing. The available data for elastic EM deuteron form factors  $A(Q^2)$  and  $B(Q^2)$  in the range  $2 - 6 \text{ GeV}^2$  ( $50 - 150 \text{ fm}^{-2}$ ) has been shown to strongly favor the present result over a variety of other approaches.<sup>12</sup> The  $\gamma\pi\rho$  mechanism for vector meson electroproduction has been treated in a closely related way.<sup>16</sup>

#### 4 Vector Meson Role in the Pion Form Factor

An important question is the size of intermediate state meson mechanisms or meson loop corrections. When the composite and extended structure of the meson modes is accounted for, the distributed vertex functions tend to leave meson loops less of a role than for models or effective field theories built on point coupling. Studies of the  $\rho - \omega$  mass difference with  $\bar{q}q$  composite pion loop dressing<sup>10,11</sup> confirm this. Here we outline an analysis of the role for the  $\rho$  in the space-like pion charge form factor. The dressed photon-quark vertex  $\Gamma_\nu(q; Q)$  can be separated (non-uniquely) into a  $\rho$  pole or resonant piece (which is transverse) and a background or non-resonant piece (which is both longitudinal and transverse). That is,<sup>2</sup>

$$\Gamma_\nu(q; Q) = \Gamma_\nu^{nr}(q; Q) - \Gamma_\mu^\rho(q; Q) \frac{T_{\mu\sigma}(Q)}{Q^2 + m_\rho^2(Q^2)} \Pi_{\sigma\nu}^{\rho\gamma}(Q). \quad (11)$$

The  $\rho\gamma$  polarization tensor is given, in abbreviated notation, by

$$\Pi_{\sigma\nu}^{\rho\gamma}(Q) = \text{Tr} [\bar{\Gamma}_\sigma^\rho(-Q) S \Gamma_\nu(Q) S]. \quad (12)$$

From Eq. (11), the pion charge form factor takes the form

$$F_\pi(Q^2) = F_\pi^{GIA}(Q^2) + \frac{F_{\rho\pi\pi}(Q^2) \Pi_T^{\rho\gamma}(Q^2)}{Q^2 + m_\rho^2(Q^2)}. \quad (13)$$

With the Ball-Chiu Ansatz used for  $\Gamma_\nu^{nr}$ , the resulting generalized impulse approximation (GIA) has been found to be phenomenologically successful in describing the spacelike  $F_\pi(Q^2)$ . A persistent result is that 85 – 90% of the charge radius is naturally explained.<sup>5</sup> The  $\rho$  resonant term is obviously necessary near the timelike pole. However the relative contribution of the two terms in the spacelike region, depends upon how the spectral strength of the underlying  $q\bar{q}$  scattering kernel is divided into pole and background. While the individual terms depend upon choice of interpolating field, the summed contribution to an S-matrix element should not.

In the limit of a point coupling model with structureless hadrons, the elements of Eq. (13) become  $F_\pi^{GIA}(Q^2) \rightarrow 1$ ,  $F_{\rho\pi\pi}(Q^2) \rightarrow g_{\rho\pi\pi}$ , and  $\Pi_T^{\rho\gamma}(Q^2) \rightarrow -Q^2/g_V$ . This produces the VMD empirical form

$$F_\pi(Q^2) = 1 - \frac{g_{\rho\pi\pi} Q^2}{g_V (Q^2 + m_\rho^2)}. \quad (14)$$

This implies  $r_\pi^2 \sim 6g_{\rho\pi\pi}/(m_\rho^2 g_V)$ , and with universal vector coupling, produces  $r_\pi^2 \sim 0.4 \text{ fm}^2$ , most of the experimental value 0.44 fm<sup>2</sup>. Given 85% of  $r_\pi$  from the GIA, is the  $\rho$  contribution small enough in the present QCD-modeling approach? Electromagnetic gauge invariance requires the  $\rho\gamma$  mixing amplitude  $\Pi_T^{\rho\gamma}(Q^2)$ , and hence the pole term of  $F_\pi(Q^2)$  in Eq. (13), to vanish at  $Q^2 = 0$ . Asymptotic freedom ensures  $\Pi_T^{\rho\gamma}(Q^2)$  vanishes at large spacelike- $Q^2$ . We use the representations  $F_{\rho\pi\pi}(Q^2) = g_{\rho\pi\pi} f_{\rho\pi\pi}(Q^2)$  and  $\Pi_T^{\rho\gamma}(Q^2) = -Q^2 \hat{f}_{\rho\gamma}(Q^2)/g_V$ , where  $f$  and  $\hat{f}$  depend on meson substructure dynamics. The  $\rho$  contribution to  $r_\pi$  from Eq. (13) is

$$(r_\pi^{pole})^2 = r_\pi^2 - (r_\pi^{GIA})^2 = 1.2 f_{\rho\pi\pi}(0) \hat{f}_{\rho\gamma}(0) \frac{6}{m_\rho^2}, \quad (15)$$

where we have used the empirical result  $g_{\rho\pi\pi}/g_V \sim 1.2$ . From Fig. 2,  $f_{\rho\pi\pi}(0) \approx 0.5$  and our calculation of  $\Pi_T^{\rho\gamma}(Q^2)$  from Eq. (12) gives  $\hat{f}_{\rho\gamma}(0) = 0.65$ . This yields  $(r_\pi^{pole})^2 = 0.16 \text{ fm}^2$  in contrast to the VMD result of 0.4 fm<sup>2</sup>. With

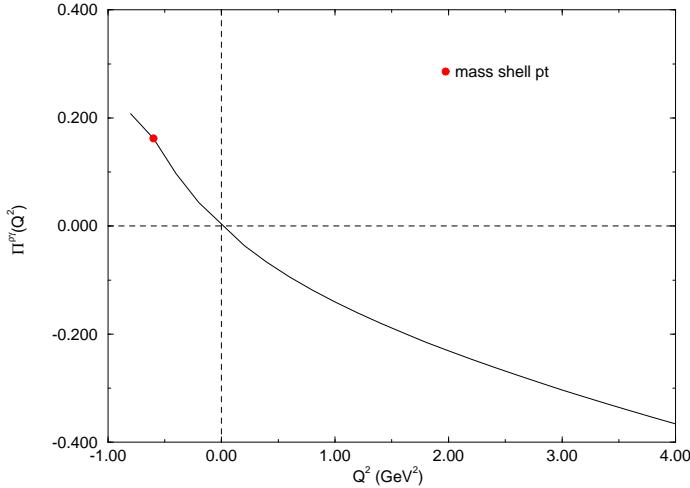


Figure 4: The transverse  $\rho\gamma$  mixed self-energy amplitude. The  $\rho$  mass-shell is indicated. The zero at  $Q^2 = 0$  preserves the massless photon.

$(r_\pi^{GIA})^2 = 0.31 \text{ fm}^2$ , the present approach gives a total of  $0.47 \text{ fm}^2$  compared to the experimental value ( $0.44 \text{ fm}^2$ ). This is obviously an overestimate leaving no room for the pion loop contribution of the expected<sup>17</sup> size. However, the main point is that a  $\rho$  contribution to the pion charge radius is a model-dependent quantity and a value much smaller than that from the simple VMD assumption is consistent with the present status of DSE-based QCD modeling of the pion.

## 5 $\rho NN$ and $\omega NN$ Couplings

The dynamical content of the simple vector meson BS amplitude of Eq. (5) has been found<sup>18</sup> to produce  $\rho NN$  and  $\omega NN$  couplings consistent with empirical values deduced from boson exchange model fits to NN data. We use a mean field chiral quark-meson model of the nucleon<sup>19,20</sup> for which the internal chiral meson modes are generated as  $\bar{q}q$  correlations. In a Euclidean metric, the relevant nucleon vector current

$$J_N^\mu(-Q) = \frac{1}{Z_N} \langle N | \int \frac{d^3 p}{(2\pi)^3} \bar{q}(p + \frac{Q}{2}) \Gamma_\nu(p; Q) q(p - \frac{Q}{2}) | N \rangle \quad (16)$$

where  $q$  is the quark field,  $Q$  is the meson momentum,  $|N\rangle$  is the static mean field nucleon state and  $\Gamma_\nu$  is the BS amplitude. The nucleon valence quark

wave function renormalization constant  $Z_N$  arises from the dynamical nature of the quark self-energy.<sup>19</sup>

At the meson mass-shell,  $\Gamma_\nu$  is normalized in the canonical way such that it is the residue of the vector  $\bar{q}q$  propagator there. However for the  $NN$  interaction, spacelike  $Q^2$  is needed. An appropriate strategy there is to use the BS eigenvalue problem  $\mathcal{K}_L(Q)\Gamma_\nu(Q) = \lambda(Q^2)\Gamma_\nu(Q)$  where  $\mathcal{K}_L$  is the BSE kernel. One can express the approach to the mass shell as  $\lambda(Q^2) = 1 - (Q^2 + M_V^2)Z_V^{-1}(Q^2)$  where  $Z_V$  is unity at the mass shell. The consistent definition of propagator for vector  $\bar{q}q$  correlations in this approach is the  $\bar{q}q$  scattering operator or T-matrix  $T = D - \mathcal{K}_L T$  where  $D$  represents the gluon 2-point function. For general  $Q^2$ , we normalize  $\Gamma_\nu$  so that the propagator for vector  $\bar{q}q$  correlations has the mode expansion form

$$\mathcal{D}_{\mu\nu}^T(q', q; Q) = \sum_n \frac{\Gamma_\mu^T(q'; Q; n) \otimes \bar{\Gamma}_\nu^T(q; -Q; n)}{(Q^2 + M_n^2)} \rightarrow \frac{i\gamma_\mu^T g(Q^2) \otimes i\gamma_\nu^T g(Q^2)}{Q^2 + M^2}. \quad (17)$$

All momentum dependence except for the standard point particle denominator has been moved into the generalized BS amplitudes. Eq. (17) also shows the point-coupling limit appropriate to the Nambu–Jona-Lasinio model<sup>21</sup> where there is no dependence on  $\bar{q}q$  relative momentum.

The form factors  $F_1$  and  $F_2$  are identified from recasting the results from Eq. (16) into the form  $J_N^\mu(-Q) = \bar{u}(\vec{p}') [i\gamma_\mu F_1(Q^2) + i\frac{\sigma_{\mu\nu}}{2M} Q_\nu F_2(Q^2)] u(\vec{p})$ . In Figs. 5 and 6 we show the results<sup>18</sup> for the  $\rho NN$  and  $\omega NN$  form factors obtained from the full BS amplitude of Eq. (5). The coupling strengths  $F_{1/2}(Q^2 = 0)$ , which are the relevant measures for the  $NN$  interaction, are given in the Table. Coupling constants from extrapolation to the mass-shell via a typical boson exchange monopole form factor with range  $\Lambda = 1.5$  GeV are shown in parenthesis. The momentum dependence and strength of the vector meson BS amplitudes are seen to be consistent with the qualitative features of the Bonn<sup>22</sup> boson-exchange  $NN$  model. A more recent value for the poorly-determined  $g_{\omega NN}$  is  $7 - 10.5$  from analysis<sup>23</sup> of pion photoproduction on the nucleon. Note that since this nucleon model overestimates the magnitude of the magnetic moments by  $\sim 15\%$ ,<sup>20</sup> one may expect at least this amount of uncertainty here also.

## 6 Summary

Since the parameters in this approach have been previously fixed through the requirement that soft chiral quantities such as  $m_{\pi/K}$ ,  $f_{\pi/K}$  and charge radii  $r_{\pi/K}$  be reproduced, the meson couplings discussed here have been produced

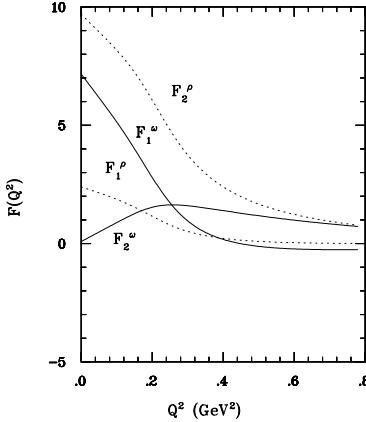


Figure 5: Vector and tensor form factors using the three covariant form of the BS amplitude.

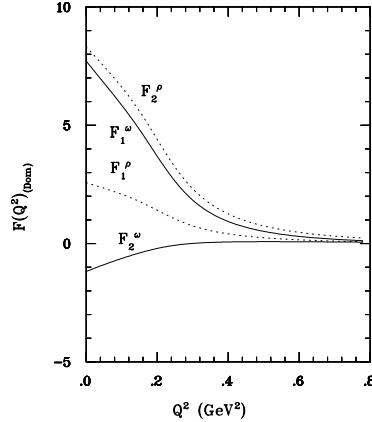


Figure 6: Only the dominant BS covariant amplitude is used here.

Table 2: Coupling constants and coupling strengths for  $\rho NN$  and  $\omega NN$

	Prediction	Empirical-OBE <sup>22</sup>
$F_1^{\omega}(0)$ ( $g_{\omega NN}$ )	7.2 (9.9)	11.7 (16)
$F_2^{\omega}(0)$ ( $f_{\omega NN}$ )	0.08 (0.11)	0 (0)
$F_1^{\rho}(0)$ ( $g_{\rho NN}$ )	2.4 (3.3)	2.6 (3.5)
$F_2^{\rho}(0)$ ( $f_{\rho NN}$ )	9.7 (13.3)	16.1 (22)
$\kappa_{\rho} = \frac{f_{\rho NN}}{g_{\rho NN}}$	4.0	6.1
$\kappa_{\omega} = \frac{f_{\omega NN}}{g_{\omega NN}}$	0.01	0
$\frac{\kappa_{\rho}}{\kappa_{\gamma}}$	1.09	1.78

without adjusting parameters. The results imply that this present approach to modeling QCD for low-energy hadron physics can capture the dominant infrared physics. We expect that the large momentum behavior of form factors such as  $\gamma\pi\pi$  and  $\pi\gamma\gamma$  will require attention to more detailed aspects of the dynamics.

### Acknowledgments

Thanks are due to A. W. Thomas for generous hospitality and the organization of a fine working environment for the Workshop. This work is partly supported by the US National Science Foundation under grant No. PHY97-22429.

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